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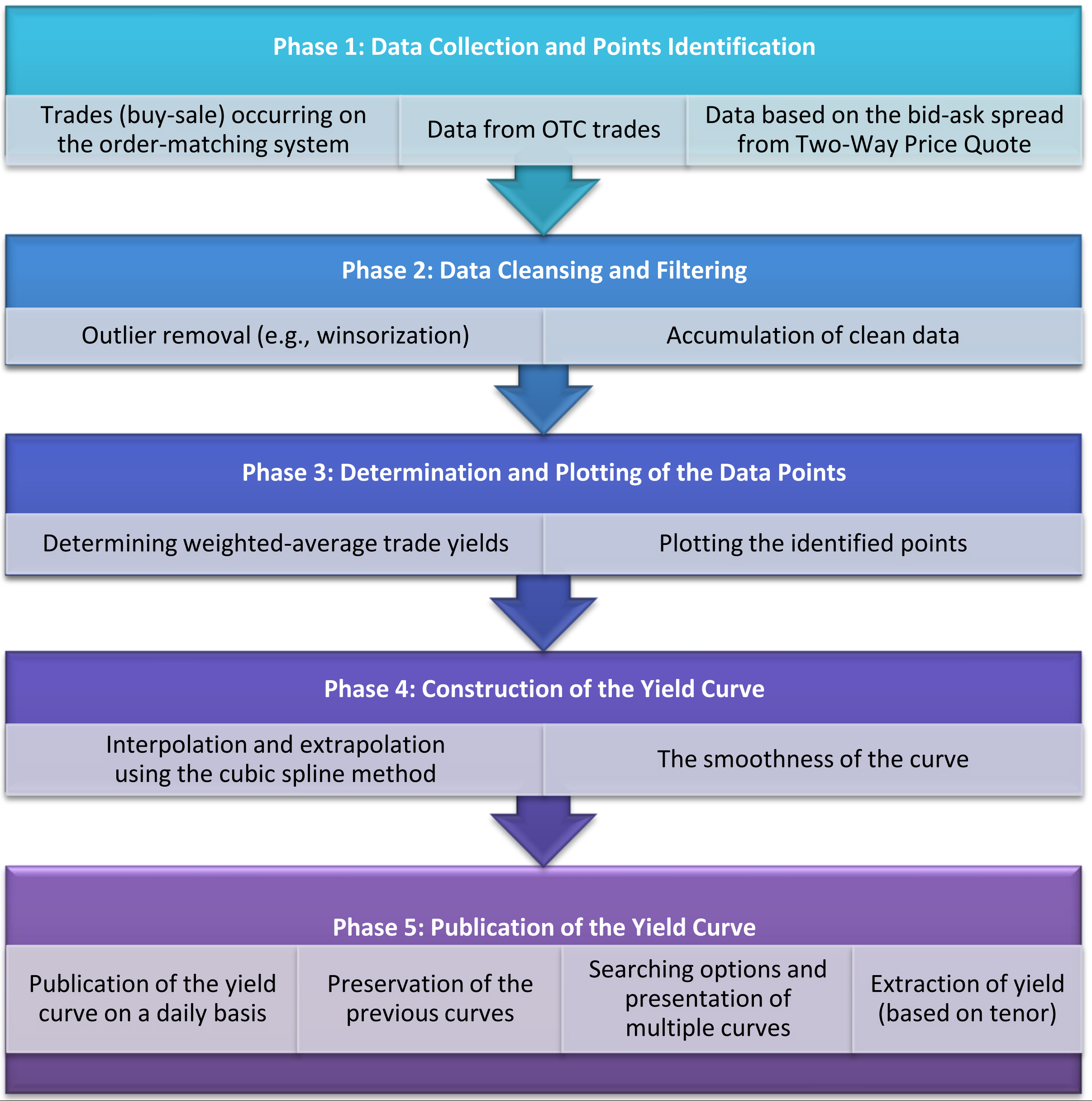
The Secondary Market Yield Curve of Bangladesh Government Securities

Debt Management Department
Bangladesh Bank

The Process Flow of the Construction of the G-Sec Yield Curve



1. Introduction

A yield curve is a graphical representation of the yields or returns on debt instruments for a range of tenors. It shows the yield investors expect to earn if they invest for a given period of time. The graph displays a type of fixed-income securities' yield on the vertical axis and the tenor (i.e., the residual time to maturity) on the horizontal axis. The shapes of the curve usually indicate different points of the business cycle; however, it is typically upward sloping.

The shapes of the curve provide the investors an indication of the probable future course of interest rates. An upward-sloping curve would mean that longer-term securities would offer a higher yield, whereas an inverted curve would show that short-term securities would have a higher yield. Apart from the shape, the steepness of a curve indicates the deviation between shorter and longer-term rates. The steeper the upward sloping curve is, the wider the difference between the rates and the higher is the profit. A flat or downward sloping curve, on the other hand, indicates otherwise.

The curve can also indicate to the investors whether a security is temporarily overpriced or underpriced. If an instrument's rate of return lies above the yield curve, this indicates that the security is underpriced; and, if the rate of return lies below the yield curve, the instrument is overpriced. Therefore, the subsequent trading of those overpriced and underpriced securities leads to a market with greater efficiency.

This concept paper describes a model of the detailed construction mechanism of a yield curve. The model described in this paper takes traded yields, as well as indicated yields (based on bid-ask spreads) for available tenors as input and generates the curve through interpolation and curve fitting while focusing on minimizing the error between traded and model prices.

2. Securities to be Considered for the Curve Construction

For the construction of the secondary yield curve of government securities (G-Sec), primarily, the benchmarked G-Sec declared (and updated from time to time) by Bangladesh Bank (BB) shall be considered. Other instruments, showing potentially a higher level of liquidity, could also be considered for the pool of securities. The securities considered for the construction of the curve will be evaluated on a periodic basis.

3. The Construction Process of the Yield Curve

Phase-1: Data Collection and Points Identification

The construction mechanism will entail tenor-wise inputs of yields from the preceding business day. Three types of inputs have been considered in constructing the curve under this mechanism. Those are the following:

A) Trades (Buy-Sale) Occurring on the Order-Matching System (OMS)

On each business day, a number of trades occur on the secondary order-matching system (OMS). The data of the trades occurring on the BB's System as well as DSE and CSE trading Platform will be accumulated as a type of real trading data and subsequently be considered for the pool of inputs under this mechanism.

B) Data from Over-the-Counter (OTC) Trades

As we know, the fixed-income securities market is usually a dealer-dominated market. This essentially leads to the majority of the trades being one on the over-the-counter (OTC) basis. Therefore, apart from the trades occurring in the OMS, the OTC trading data will also be collected for the construction of the secondary yield curve of G-Sec.

C) Data Based on the Bid-Ask Spread from Two-Way Price Quote

Globally, the majority of the fixed-income securities do not get traded on a frequent basis in the secondary market. This phenomenon potentially leads to the stale pricing of those instruments, and consequently, the market value of those instruments becomes complicated to determine.

Therefore, to incorporate the potentially available market liquidity in the construction process, the bid and ask quotes submitted on the order-matching system available at the end of the trading hour on the preceding business day will also be collected from the order books. As at present, the primary dealers of the G-Sec market are instructed to post two-way (bid and ask) quotes on each trading day against the benchmarked securities, a large number of inputs should be expected from this source.

Calculate the midpoint weighted-average Yield (WAY) for each securities in the following manner:

a. Collection of Buy side Order Yield and Sell side Order Yield
    i. Calculate spread between best Buy side Order Yield with best Sell side Order Yield and again calculate spread between next best Buy side Order Yield with next best Sell side Order Yield and so on;
    ii. Consider the orders having spread between the bids and offers upto 200 basis points;
    iii. Calculate Buy side WAY and sell side WAY of previously accepted order excluding the outliers.
b. Calculate the midpoint WAY [(Buy side WAY + Sell side WAY)/2]. Subsequently, the midpoints yields in between those calculated weighted-average bids and weighted-average ask have been used in deriving the corresponding implied yields.

At present, the calculated midpoint yield of all the benchmarked securities has been considered. In the future, the calculated midpoint yield will only be used for tenor points where no actual trade (OTC & Order Matching) exists.

Phase-2: Data Cleansing and Filtering

A) Outlier Removal

In this phase, the data accumulated from Phase-1 will go through a filtering process (e.g., winsorization) to avoid the potentially skewed data impacting the construction process. As we know, in spite of applying various rigorous selection criteria, the traded and implied yields of a few instruments may yet deviate significantly. To thwart the noise in the yield curve estimation process by minimizing the impact of noise trades, we have to eliminate the outliers from our sample.

After collecting data from the midpoint of WAY from two-way price quote, actual traded data of OTC and Order Matching (OM), the following process is being used in order to remove the outlier data:

a. Calculate average of yield of each securities
b. Deduct average yield from individual yields $(x_i - \bar{x})$
c. Square of each calculated deviations $(x_i - \bar{x})^2$
d. Eliminate the yield for which calculated squared deviation exceeds 0.5

However, in future, the below mentioned procedure may be followed to remove the outlier data:

Securities with yields that diverge by more than 02 standard deviations [as per Nymand-Andersen (2018)] from the average in each maturity bracket (0-2 years, 2-5 years, 5-8 years, 8-10 years, 10-15 years, and 15-20 years) should be considered as outliers, and those outliers will be removed from our data pool.

In each of these brackets, the average yield and standard deviation will be calculated. This procedure should be iterated to reduce the sensitivity of the analysis to potentially large outliers eliminated in the first step that could have distorted the average yield level and the standard deviation. Based on Nymand-Andersen (2018), the following figure illustrates the process of outlier elimination:

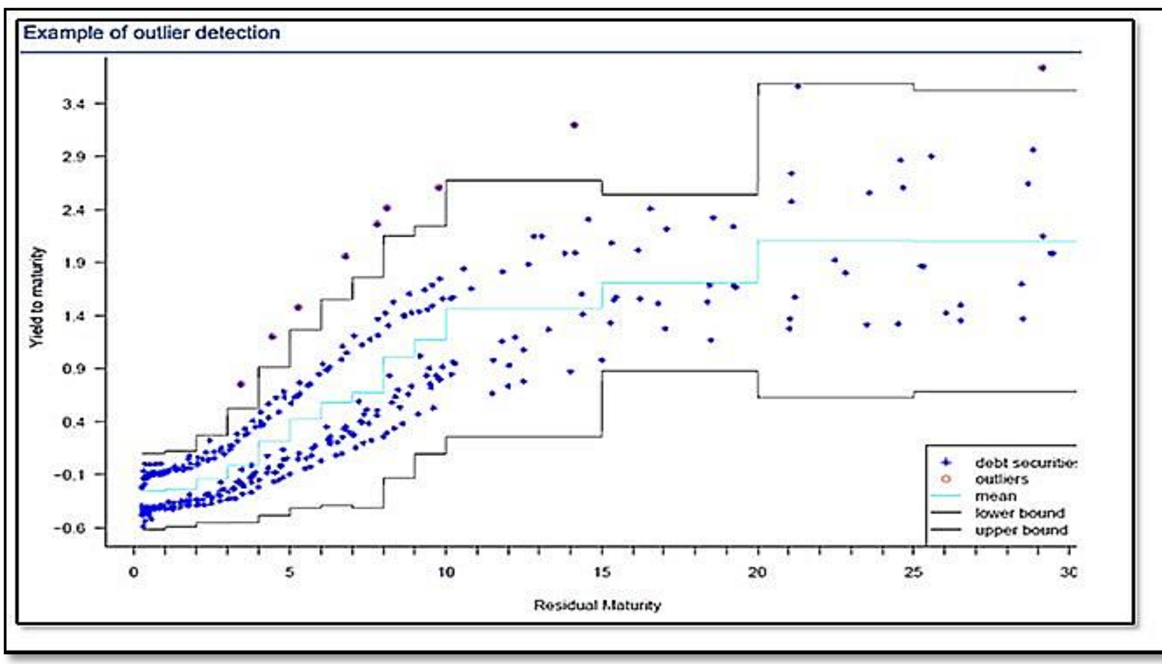


Figure: Data Cleansing [Nymand-Andersen (2018)]

B) Accumulation of Clean Data

After the removal of outliers, the tenor-yield coordinates will be accumulated. This accumulated clean data pool will then be considered for the construction process. These points accumulated in the pool will then be used in determining the relevant yields (based on tenor).

Phase-3: Determination and Plotting of the Data Points

A) Determining Weighted-Average Yields of Each Securities

After eliminating the outliers in Phase-2, the remaining data will be considered for yield curve construction. In this case, each tenor with multiple traded and implied (quote-based) yields will be used to determine the weighted-average yields of each securities.

B) Plotting the Identified Tenor-Yield Points

The calculated weighted average yield points of each securities mentioned in the Phase 3 (A) will be plotted as per tenor. The standard tenor points for the curve is 06 month time interval ranging from 06 months to 20 years. Simple interpolation and extrapolation method used from the nearest yield of each remaing maturity to calculate the yields of each standard tenor. As per the norm of constructing the term structure of interest rates, these tenor points will be portrayed on the x-axis. The yields of the G-Sec will be portrayed on the y-axis of the graph.

Phase-4: **Construction of the Yield Curve**

After plotting the data points on the two-dimensional surface, the intermittent portions of the intended curve will be constructed via interpolation and extrapolation. The curve has been constructed and published in the above mentioned process but in the future Cubic spline method, as described below may be used to smooth the curve.

To construct the yield curve, as originally proposed by McCulloch (1975) as well as per Pienaar and Choudhry (2010), we will be employing the cubic spline method. We can expect a curve constructed using the cubic spline approach by fitting the data will offer a significantly smoother curve for the yields of G-Sec in accordance with their corresponding term discount factors.

A) Interpolation and Extrapolation Using the Cubic Spline Method

A spline is a function that is constructed piecewise from polynomial functions. Here, we can envisage the polynomials as pieces of the term structure of interest rates. Each term structure segment is marked off by a set of price and maturity pairs. Whole sections can be marked off by a knot at a location in the term structure paired data. Knots are most commonly placed at quartiles to put more knots where data are clustered close together [Foote (2018)]. A different polynomial function is estimated for each range and domain of data between each knot: this is the spline. In general, a polynomial is an expression that looks like the following:

$$f(x) = a_0x^0 + a_1x^1 + a_2x^2 + \ldots + a_px^p;$$

Where the a's are constant coefficients, and $x^0 = 1$ and $x^1 = x$.

- If p =0, then we have a constant function,
- If p =1, we have a linear function,
- If p =2, we have a quadratic function,
- If p =3, we have a cubic function, and so on.

Therefore, in our term structure, we will be using a cubic function to build a regression spline. So, a cubic spline is a spline, which is constructed of piecewise third-order polynomials that pass through a set of i number of control points. The second derivative of each polynomial is commonly set to zero at the endpoints since this provides a boundary condition that completes the system of $m - 2$ equations. This produces a so-called "natural" cubic spline and leads to a simple tri-diagonal system, which can be solved easily to give the coefficients of the polynomials [Weisstein (2004)]. Following Bartels et al. (1998), the i th piece of the spline of third-order would be,

$$Y_i(t) = a_i + b_it + c_it^2 + d_it^3 \quad (1)$$

Where t is a parameter $t \in [0,1]$ and $i = 0, \ldots, n - 1$.

The particulars and the process of extracting the values of the coefficients in calculating the values of the dependent variable are mentioned in the appendix.

B) The Smoothness of the Curve

As stated in the earlier sub-section, the intended curve will be constructed using a polynomial of the 3rd degree under the spline method. This will be integral in ensuring that the yields calculated from the curve will closely represent the market rate for each corresponding tenor.

Phase-5: Publication of the Yield Curve

A) Publication of the Yield Curve on a Daily Basis

After extrapolation and interpolation, on the curve, the tenor-yield (x, y) data will be visible for each monthly interval (between 0 and 20 years). The yield curve constructed following the above method is published on each business day. Upon successful publication of the curve, it will be used for the valuation of the relevant instruments.

B) Preservation of the Previous Curves

The data points of each business day's curve should be treated as "non-destroyable" for data preservation purposes. The data of the previous curves will be stored in the portal and be readily available for the users.

C) Search Options and Presentation of Multiple Curves

Alongside the published yield curve, a search option should be available for the users to look for the curves of the preceding business days. The previous days' curves should remain available should a user chooses to compare multiple curves from several trading dates (by offering an option to choose from a graphical calendar embedded in the display portal). Therefore, the platform should be equipped to present a minimum of 3 (three) different business days' curves.

D) Extraction of Yield (Based on Tenor)

A calculation tool should be embedded and offered in the portal to the users to instantly calculate the yields based on tenors. In this tool, by providing the tenor as an input (up to 4 decimal points), a user should be able to extract the respective yields (up to 4 decimal points) and vice versa.

Bibliography

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Appendix

Cubic splines are implemented in the Wolfram Language as BSplineCurve. Consider 1-dimensional spline for a set of $n+1$ points $(y_0, y_1, \ldots.., y_n)$. Following Bartels et al. (1998), let the i th piece of the spline be represented by,

$$Y_i(t) = a_i + b_i t + c_i t^2 + d_i t^3 \tag{1}$$

Where t is a parameter $t \in [0,1]$ and $i = 0, \ldots, n-1$.

Now, following Weisstein (2004),

$$Y_i(0) = y_i = a_i \tag{2}$$

$$Y_i(1) = y_{i+1} = a_i + b_i + c_i + d_i \tag{3}$$

Taking the derivative of $y_i(t)$ in each interval then gives [Weisstein (2004)],

$$Y_i^{'}(0) = D_i = b_i \tag{4}$$

$$Y_i^{'}(1) = D_{i+1} = b_i + 2c_i + 3d_i \tag{5}$$

Solving equations from (2) to (5) for a_i, b_i, c_i and d_i gives,

$$a_i = y_i \tag{6}$$

$$b_i = D_i \tag{7}$$

$$c_i = 3(y_{i+1} - y_i) - 2D_i - D_{i+1} \tag{8}$$

$$d_i = 2(y_i - y_{i+1}) + D_i + D_{i+1} \tag{9}$$

Now require that the second derivatives also match at the points; therefore,

$$Y_{i-1}(1) = y_i \tag{10}$$

$$Y_{i-1}^{'}(1) = Y_i^{'}(0) \tag{11}$$

$$Y_i(0) = y_i \tag{12}$$

$$Y_{i-1}^{''}(1) = Y_i^{''}(0) \tag{13}$$

For interior points, as well as that the endpoints satisfy,

$$Y_0(0) = y_0 \tag{14}$$

$$Y_{n-1}(1) = y_n \tag{15}$$

This gives a total of $4(n-1)+2 = 4n-2$ equations for the $4n$ unknowns. To obtain two more conditions, require that the second derivatives at the endpoints be zero, so,

$$Y_0^{''}(0) = 0 \tag{16}$$

$$Y_{n-1}^{''}(1) = 0 \tag{17}$$

Rearranging all these equations, according to Bartels et al. (1998), leads to the following symmetric tri-diagonal system.

$$\begin{bmatrix} 2 & 1 & & & & \\ 1 & 4 & 1 & & & \\ & 1 & 4 & 1 & & \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots \\ & & & 1 & 4 & 1 \\ & & & & 1 & 2 \end{bmatrix} \begin{bmatrix} D_0 \\ D_1 \\ D_2 \\ \vdots \\ D_{n-1} \\ D_n \end{bmatrix} = \begin{bmatrix} 3(y_1 - y_0) \\ 3(y_2 - y_0) \\ 3(y_3 - y_1) \\ \vdots \\ 3(y_n - y_{n-2}) \\ 3(y_n - y_{n-1}) \end{bmatrix} \tag{18}$$

If the curve is instead closed, the system becomes:

$$\begin{bmatrix} 2 & 1 & & & & 1 \\ 1 & 4 & 1 & & & \\ & 1 & 4 & 1 & & \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots \\ & & & 1 & 4 & 1 \\ 1 & & & & 1 & 4 \end{bmatrix} \begin{bmatrix} D_0 \\ D_1 \\ D_2 \\ \vdots \\ D_{n-1} \\ D_n \end{bmatrix} = \begin{bmatrix} 3(y_1 - y_n) \\ 3(y_2 - y_0) \\ 3(y_3 - y_1) \\ \vdots \\ 3(y_n - y_{n-2}) \\ 3(y_0 - y_{n-1}) \end{bmatrix} \tag{19}$$